

EQIP Review Feedback



Lesson/Unit Name: Factor, Expand, and Combine Like Terms

Content Area: Mathematics

Grade Level: 7

Overall Rating:

E/I

Exemplar if Improved

Dimension I – Alignment to the Depth of the CCSS

<p><i>The lesson/unit aligns with the letter and spirit of the CCSS:</i></p> <ul style="list-style-type: none"> ✓ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning. ✓ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. ✓ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS. 	<p>The standard for this lesson is 7.EE.A.1: "Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients." The activities in the lesson are representative of the depth of understanding expected in this standard. From the warm-up/motivation activity on p. 2 to the end of the lesson, the activities encompass the full set of rational numbers, including having to add, subtract, expand, and/or factor expressions that have negative and positive integer, fraction, and decimal coefficients. This is evidenced by the content of the cards on Attachments 1 and 2, and also the content for Attachments 5 and 6.</p> <p>The Standards for Mathematical Practice are directly referenced for Activity 1 (p. 3), Activity 2 (pp. 4-5) and Activity 3 (pp. 6-7). They are handled in a grade-appropriate way and are well connected to the content being addressed. The SMPs are explained in relation to the activities. Specific language of the explanations indicates the relationship between student behaviors and the intended proficiency/purpose of each activity. The SMPs are also identified in the activity narrative to remind teachers of where they should be seeing each SMP occur in connection with mathematical content. You may want to consider adding them to the Warm-up Activity on p. 2.</p> <p>The lesson leans more to procedural skill due to the nature of the standard. However, the lesson does provide students with a balance of procedure in an engaging way with the card activities and deep understanding of the impact of properties of operations as strategies. You may want to consider that conceptual models, such as two-color counters, algebra tiles, number lines, etc., may still be helpful for those students who choose to use them to support their understanding, especially when reinforcing understandings of simplifying expressions with integer coefficients which would occur in a prior lesson.</p>
<p>Rating: 3 – Meets most to all of the criteria in the dimension</p>	

Dimension II – Key Shifts the CCSS

<p><i>The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:</i></p> <ul style="list-style-type: none"> ✓ Focus: Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades. 	<p>This lesson focuses on standard 7.EE.A.1 and is noted as major work for the teacher on page 1. The lesson does not hold students responsible for material from later grades. The lesson is appropriately focused on the major work of the grade and makes visible connections to previous work in the grade.</p> <p>The Coherence standards from both the previous grade level and the current grade level are listed for the teacher on pp. 1-2. Consider adding the full text of the standards so that 7th grade teachers will know the depth of the understanding across, within the grade, and the foundation being laid for the next grade. It may be appropriate to add 6.EE.A4: "Identify when two</p>
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✓ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.

✓ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

- **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
- **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
- **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for." Lastly, to show the importance of 7.EE.A.1 for 8th grade, you may also want to include 8.EE.C.7b: "Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms."

You may want to also consider how to help the teacher with how to directly connect these Coherence standards, especially 6.EE.A.3 and 7.NS.A.1-2, to the Warm-up and Activity 1 on pp. 2-4. Do your initial activities intentionally enable students to make visible connections to the prior knowledge gleaned from 6.EE.A.3, 6.EE.A.4 and 7.NS.A.1-2 to 7.EE.A.1? What are the necessary prerequisite skills in order for students to be successful in this lesson. For example, it is assumed that students have already mastered all operations with rational numbers beyond integers, and simplifying expressions with integer constants and coefficients before approaching this lesson with rationals and rational coefficients. That would be a reasonable progression of learning as students do not operate with negatives until 7th grade. If this is so, then we might question the content of the warm-up to identify terms as constants or "variable terms", to identifying like terms, to joining like terms and then working with the distributive property. If students identified like terms in 6.EE.A.4 and have already worked with integer terms in 7th grade, this part of the warm-up would not be warranted. It may be good to say that these are prerequisite understandings and skills for this lesson. For those students who may not remember why "x's" cannot be joined with "y's" or constant terms, you may want to consider preparing ahead of time for a quick refresher that is visual and conceptual. You do this later in the lesson with the books and apple example, but it may be warranted during the warm-up.

While the Rigor shift is present, it is not as strong as it could be. The activities do not provide opportunities for students to independently apply mathematical concepts in real-world situations, to solve challenging problems with persistence, or to choose and apply an appropriate model or strategy to new situations. Instead, the lesson activities emphasize procedural skill and fluency by providing ample opportunities for students to choose, discuss, explain and defend strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Students are able to understand and practice core calculations and mathematical procedures quickly and accurately. Conceptual understanding is evidenced in the lesson as students are able to develop conceptual understanding through tasks, brief problems, questions, and multiple representations. The lesson activities also provide students with chances to work independently as well as to write and speak about their understanding.

The Rigor components listed on p.2 of the plan accurately represent the Procedural Skill expected in the activities. Please consider rewording the following on that same page:

- The Conceptual Understanding described does not accurately describe this lesson. In the lesson students are not solving equations. Instead, they are simplifying and factoring expressions.

- Modeling/Application: While students are asked to use procedural skills to join and factor constant terms and rational coefficients for like terms, they are not actually applying these skills in a real-world context nor are they asked to create mathematical models/expressions from contextual situations. Due to the mathematics inherent in the standard and lesson, not

all lessons will have this component, but you may consider somehow connecting and reinforcing the verbal descriptions of expressions established in 6.EE.A.2a: "Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as $5 - y$." At what level of accuracy and precision (SMP6) do you expect the students to read aloud the expressions in the lesson?

Rating: **3 – Meets most to all of the criteria in the dimension**

Dimension III – Instructional Supports

The lesson/unit is responsive to varied student learning needs:

- ✓ Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.
- ✓ Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.
- ☐ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.
- ✓ Addresses instructional expectations and is easy to understand and use.
- ☐ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.
 - Supports diverse cultural and linguistic backgrounds, interests and styles.
 - Provides extra supports for students working below grade level.
 - Provides extensions for students with high interest or working above grade level.

A unit or longer lesson should:

- ☐ Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
- ☐ Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
- ☐ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.
- ☐ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

The lesson activities include clear and sufficient guidance to support teaching and learning of the targeted standards. Each component of the lesson provides an explanation of teacher and student actions and the learning purpose. The directions and expectations are easy to understand and will help guide teachers through the lesson. The lesson provides teachers with questions to ask students to help them through thought provoking tasks.

The understanding of a "coefficient" should have already been developed in 6th grade during 6.EE.A.2b: "Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms." For additional clarification, the other grade-level standards that the activities incorporate could be mentioned to highlight within-grade coherence. On p. 3 in the Vocabulary Builder section it is noted that "As the activity progresses, more specific vocabulary should be developed." The vocabulary "variable term" is used in the warm-up and throughout the activities. Consider formally defining a "variable term" as that which has a "coefficient" and it is the variable itself that defines like terms. The coefficient tells us the value of the variable. For example, $-1/2a$ is "the opposite of $1/2$ of a ".

Is the work mat (Attachment 3) to be oriented horizontally, vertically or does it matter? If you want a specific orientation, perhaps there should be some instruction on this. The work mat (Attachment 3) seems somewhat restricted in its set progression of operations. Should there be some attention paid to the fact that different solutions can be obtained depending on the orientation of the mat and the order of the cards? This might be an interesting variation for exploration by students.

With regards to the cards, I copied Attachment 2 and replicated the instructions for Activity 2. Because there are now four variables (a , b , x , and y) on the cards, there was only one time that I randomly chose four cards that needed any simplification whatsoever. Although it is an important understanding when an expression cannot be simplified, I think you desired more outcomes that actually require simplification. Your question on p. 6 regarding joining $1/3a$ and $0.8a$ is very good, but that situation never happened during several iterations of drawing four cards. Perhaps you can join the cards from the warm-up Attachment 1 that have x 's and y 's with the cards from Attachment 2 to create more possibilities for x , y and constant like terms? That is just one idea and you may think of others.

Even though the lesson activities may stimulate interest and elicit mathematical thinking among students through relevant, thought-provoking questions, the lesson does not consistently engage students in productive struggle. More guidance for new teachers that provide opportunities for

students actually discuss their ideas for simplification and engage in justification and argument (SMP3) might be helpful.

On p. 8, there is a list of suggested interventions and enrichments. Consider adding more guidance for a new teacher such as the following:

- You suggest to first start with all like terms for struggling learners. How many like terms? What should this progression to mixing in unlike terms look like? How many of what type to be introduced in each sequence?
- This lesson could be strengthened by including more supports for students who are English Language Learners or students with disabilities. Some guidance around how to respond if students are not able to identify the like terms or do not understand how to combine them would strengthen this lesson. ELL students (really all students) need many visuals associated with the conceptual meaning of mathematical vocabulary. Consider giving them an example with a visual, but then letting them create a different example on their own. If we create the word wall as teachers, students are not engaging in the struggle to understand the term and create their own meaning.

There was some discussion among the multiple reviewers as to whether or not this lesson fell into the category of a "longer lesson". It was decided that it did not, although the consensus of the reviewers was that the lesson author should consider these criteria in developing the progression of learning to occur in lessons before and after this lesson.

Rating: 2 – Meets many of the criteria in the dimension

Dimension IV – Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:

- ✓ Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
- ✓ Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
- Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

A unit or longer lesson should:

- Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.

There are documented opportunities throughout the lesson to gather information regarding the current status of student understanding. These come in the form of a formative warm-up/motivation, questions for the teacher to pose during the activities in the Group Discussion sections, an Exit Ticket for Activity 2, a suggested Formative Assessment idea for Activity 3, and a 3-2-1 Closure for Activity 3. These activities also serve as student self-assessment measures.

The above are a great start for ongoing assessment. Following are some other suggestions.

- You may want to add more suggestions to the teacher for how to analyze certain student errors or misconceptions that may appear. For example, some students may still continue to struggle with the operations with rational numbers which leads to the inability to simplify expressions accurately. What should a teacher do then? Could there have been a pre-assessment of rational number operation skills before this lesson? What scaffolds and supports could be put in place for those students?
- The Exit Ticket for Activity 2 has no constants and two sets of like terms. Did the students practice this particular situation enough during the Activity 2 to be successful on the Exit Ticket?
- The Formative Assessment at the end of Activity 3 is good. What should the teacher look for in the student results. After the groups check each other's problems, what feedback would they give to each other and what support can we give students as teachers to guide this feedback?
- The lesson activities provide teachers with some answer keys that are not truly sufficient for all teachers, especially new teachers or those for whom the content is new. The lesson would be stronger with additional guidance for interpreting student performance. Consider providing teachers with rubrics and more guidance on how to analyze student work and Group

	Discussion answers. This includes providing help for other teachers on how to identify common misconceptions and error patterns and also how to ask the needed probing questions so that students can analyze and correct their own errors.
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Rating: 2 – Meets many of the criteria in the dimension

Summary Comments

Overall, this is a very strong lesson. It shows a clear understanding of the Common Core State Standards and the work that they demand of students. It is well aligned, gives clear guidelines, and is mathematically sound. This lesson is on its way to become a Common Core Exemplar with just a few adjustments. Consider:

- Adding more information about scaffolding for students at different levels of understanding.
- Adding the full text of the coherence standards and the additional suggested coherence standards on p. 2 of this review.
- Stating that teachers might consider pre-assessing the knowledge and understandings retained from the 6.EE and 7.NS.1-2 and then providing ideas for teachers to support and scaffold learning for students still struggling or dealing with misconceptions with these prerequisite standards so that they can work with the depth of 7.EE.A.1.
- Being more precise throughout the lesson with vocabulary and verbal descriptions of expressions that were already introduced in 6th grade.
- Modifying the card activity to provide more opportunities for like terms much like those in the Exit Ticket for Activity 2, including a question that requires students to join constants.
- Even though some activities rely on 7.EE.B.4, (Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities), only non-contextual problems are included. The lesson could be enhanced further with challenging, real-world problems for students to solve.
- The lesson would be stronger with additional guidance for interpreting student performance. Providing clues for teachers on identifying common misconceptions and error patterns that may arise during the lesson and how to ask guiding questions that require the students to deal with their own errors in thinking or computation. This includes ways to identify these same misconceptions and errors on assessments throughout the lesson.

Rating Scales

Rating Scale for Dimensions I, II, III, IV:

3: Meets most to all of the criteria in the dimension

2: Meets many of the criteria in the dimension

1: Meets some of the criteria in the dimension

0: Does not meet the criteria in the dimension

Overall Rating for the Lesson/Unit:

E: Exemplar – Aligned and meets most to all of the criteria in dimensions II, III, IV (total 11 – 12)

E/I: Exemplar *if* Improved – Aligned and needs some improvement in one or more dimensions (total 8 – 10)

R: Revision Needed – Aligned partially and needs significant revision in one or more dimensions (total 3 – 7)

N: Not Ready to Review – Not aligned and does not meet criteria (total 0 – 2)

Rating Descriptors

Descriptors for Dimensions I, II, III, IV:

3: **Exemplifies CCSS Quality** - meets the standard described by criteria in the dimension, as explained in criterion-based observations.

2: **Approaching CCSS Quality** - meets many criteria but will benefit from revision in others, as suggested in criterion-based observations.

1: **Developing toward CCSS Quality** - needs significant revision, as suggested in criterion-based observations.

0: **Not representing CCSS Quality** - does not address the criteria in the dimension.

Descriptor for Overall Ratings:

E: **Exemplifies CCSS Quality** – Aligned and exemplifies the quality standard and exemplifies most of the criteria across Dimensions II, III, IV of the rubric.

E/I: **Approaching CCSS Quality** – Aligned and exemplifies the quality standard in some dimensions but will benefit from some revision in others.

R: **Developing toward CCSS Quality** – Aligned partially and approaches the quality standard in some dimensions and needs significant revision in others.

N: **Not representing CCSS Quality** – Not aligned and does not address criteria.